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The Impact of Endogenously Determined Uncertainty about Future Mispricings and Investigatory Outcomes

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Abstract

This paper models market mispricings as a function of subjective predictions about the endogenously affected but always uncertain trades of other investors. Utilizing only a very general set of unrestrictive assumptions about investors and markets, a single boundary condition with just a few variables is deduced that facilitates rational analysis of the multitude of ambiguous factors and complex interrelationships that influence investment decisions. The model supplies a useful framework for understanding market cycles and other phenomena, including the neglected firm effect which is fully comprehensible within the context of logical reactions to the unknown outcome of any investment evaluation.

Introduction

Various reasons have been advanced for markets not immediately correcting deviations between prices and intrinsic values in the security markets. For instance, it has been shown that mispricings may persist due to trading expenses (Pontiff, 1996), the costs of analyzing asset values (Grossman and Stiglitz, 1980), constraints on arbitrager wealth (Schleifer and Vishny, 1992), restricted borrowing capacity (Liu and Longstaff, 2004), and motivational limitations on information dissemination (Schleifer and Vishny, 1997). In addition, Abreu and Brunnermeier (2002, 2003) have demonstrated that the possibility of a known mispricing increasing in size may have an especially important impact on the persistence of deviations between prices and intrinsic values because of the potential optimality of postponing arbitrage trades under such circumstances.

The analytical study conducted here expands on this line of research by allowing for uncertainty in all the variable distributions and parameter values in a fashion that Epstein and Ji (2013) have indicated is important in fully comprehending the true nature of security prices. A complex mathematical model of equilibrium prices under such general conditions has been previously developed by Murphy (1990), but the behavioral implications for traders in such a real world have never previously been explored. In addition, the full implications of this mathematical modeling of reality, such as with respect to explaining market anomalies, have never been deduced.

Despite incorporating all the complexity of the actual market environment, a simple equation is deduced for optimal arbitrager decision-making that creates a foundation for understanding markets associated with a multi-period model that includes multiple mispriced assets and investors with unspecified heterogeneous beliefs.

By not artificially specifying an objective function for mispricings or the variables that determine them, arbitrager behavior is realistically set to be affected by the true uncertainty of the future direction of market prices, which are endogenous to aggregated investor decisions and thus don't have a fixed probability of a persisting or growing deviation from intrinsic values. Such uncertainty can motivate investors to optimally utilize human intuition based on emotions to make investment decisions, thereby resulting in trading activities being influenced by biological factors, which themselves are impacted interactively by the changing market conditions they cause (Coates, 2012). Both rational and emotional reactions to the resulting patterns in market movements make it impossible to fully specify relevant parameter values ex-ante since the complex, varying psychological interrelationships that influence them can't be fully known in advance (Murphy, 2012). Within this context, the likelihood of arbitrage becomes a subjectively determined variable that is affected by the aggregated and often changing judgments of all investors, which result in an unknown distribution for the present and future size of a mispricing.

The model is integrated with a framework for subjectively determining whether to conduct an investment analysis, which itself has an uncertain probability of a perceived mispricing actually existing and being large enough to lead to a rational conclusion to trade the asset. Since the possibility of an investment analysis that doesn't result in a transaction is a function of the size of a mispricing, the likelihood of a failed investigation is endogenous to the market prices which determine the extent of any deviation from intrinsic value. This probability, which is therefore too uncertain to be exactly specified, is shown to have a potentially enormous impact on the market prices of assets with which investors aren't very familiar.

Investors using fundamental analysis to select the securities they trade based on their appraisals of deviations between intrinsic values and market prices can utilize the model to determine if and when to conduct a transaction, as well as to decide whether to even consider any potential investment. In addition, technical analysis, which typically only uses a reduced form of the full structural effect of human nature on investment asset prices, may be enhanced by incorporating the implications of this research. In particular, a complete recognition of the interrelationships between market prices and investor psychology that impact arbitrage behavior which is modeled here can assist investors in developing and evaluating trade-timing signals than is possible through mere knowledge of past price patterns and statistical correlations.

Numerous testable hypotheses, which shed new insights on observable market characteristics appearing to be anomalies, are deduced from the analytically derived boundary condition for optimally triggering arbitrage. For instance, the small firm effect relating to higher measured risk-adjusted returns on stocks with lower market capitalizations may be fully explained by the expected trading profits from arbitrage having to cover the expenses of potentially failed investigations of such investment opportunities. They therefore have a much wider range of possible mispricings that would tend to move more with overall market extremes, thereby causing such assets to have higher true systematic risk than is normally measured using past data. In addition, the theory yields hypotheses on the reasons for market cycles that can be empirically tested with an analysis of the relationship between the model's variables. Simple introductory tests provide supporting evidence on those causes.

In Section I, a model of optimal arbitrager trading tactics is developed. Some of the theory's implications are explained in Section II with respect to market cycles, special small firm effects, and the impact of institutional performance measurement schemes. The work is summarized in Section III.

I. The Model Assumptions and General Deductions

The incentives arbitragers have to wait for a better transaction opportunity can be shown with the following simple analytical model, which relies on four assumptions that are grounded in the reality of human behavior and actual market characteristics. These assumptions generalize to most situations encountered in investments.

Assumption #1:

Assume that investors act as if they are maximizing the expected value of a Von Neumann and Morgenstern (1947) function for the utility of consumption that has parameter values which vary over time. The utility function's variables, which realistically include social and psychological rewards in addition to the material consumption that motivate human behavior (Bernstein and Nash, 2008), may be affected by biology-induced psychological influences. The latter can promote systematic trends in investors' optimism or pessimism about the estimated future values of relevant factors like returns on invested wealth, which sets the boundary for spending on consumption.

Assumption #2:

Assume a risky asset d has future cash flows that aren't known with certainty and that may be estimated differently by the various investors in the market place using an unspecified combination of logical reasoning and intuition. The costs associated with evaluating the individualized utility or intrinsic value of d to a particular investor is defined as Ψ while the actual costs of a transaction in the asset equals κ, which includes explicit brokerage expenses as well as adverse pricing relating to trading pressure/spreads. These expenses of investing are specified to be a fraction of the asset's market price for a particular trading opportunity and incorporate the expected size of the position in the asset potentially determined only after a costly analysis, which has an uncertain outcome with respect to indicating any trade is optimal. The difference between an investor's appraised value of d and its market price, which is determined by normal laws of supply and demand for assets in a centralized market to establish the point where buy and sell orders exactly match, is defined by an unspecified absolute value *D* set as a fraction of the price.

Assumption #3:

Assume there is at least one other risky asset μ, which can be an entire portfolio of securities, and whose market price deviates from its intrinsic value by an uncertain amount that can generate, net of the costs involved in evaluating and trading on the opportunity, a present value of expected excess profits equal to *M* that are measured net of the investigative, transaction and other costs associated with arbitraging it. Just as for *D*, the latter amount may be heterogeneously estimated by each investor. In addition, a risk-free asset f also exists that has a known return with a zero variance over an investor's potential holding period.

Assumption #4:

Assume there are an uncertain number of investors who believe they can maximize their own utility functions by trading securities with market prices that deviate from their "true" intrinsic values which would exist if individuals' portfolio holdings truly maximized their utility functions in a world with complete information on all the relevant variable distributions. Each of these individual arbitragers, who may need to incur investigative costs Ψ to verify an initial estimate of *D* that has an unknown probability ω of being valid, is constrained in any particular period by finite wealth, borrowing limits, diversification considerations, and other restrictions to trading in finite amounts, so that transactions on any one of the three assets d, μ, or f reduces the size of the position that can be taken on the other two.

A. General Deductions Indicated by the Model Assumptions

As shown mathematically by Murphy (1990), the foregoing Assumptions #1-3 imply that an asset's market price will equal the weighted-average estimate of its perceived intrinsic value to investors, where the weights are determined by the amount invested into the asset by each investor, who establishes position sizes within the context of the overall portfolio holdings subjectively considered to be maximize the investor's utility. This derived model of equilibrium market prices indicates each value in the weighted average equals a different investor's subjective estimate of the sum of the cash flows heterogeneously expected from the asset that are discounted at the required expected return differentially determined by the investor's own situation and utility function, which may vary cross-sectionally and temporally because of factors like taxes, transaction costs, and portfolio composition. The initial portion of the proof and some important mathematics stated to be "available from the author" in the cited article are provided in the Appendix, as is an explanation for the remainder of the derived equations in an intuitively comprehensible form. These equations enable a better understanding of the variables that determine market prices, but the optimal trading behavior of investors in this realistic environment has never been examined.

When prices in a dynamic market deviate from the values that are individually appraised by each investor, those who believe the value of an asset to themselves to be higher (lower) than the market price at any moment are motivated to purchase (sell) the asset. Such trading behavior also characterizes arbitragers who, according to Assumption #4 (that was not specified in the original model but certainly consistent with it), attempt to maximize their own wealth and consumption utility by conducting arbitrage transactions on assets they perceive to have prices which deviate from the "true" intrinsic values across all investors. Those intrinsic values have previously been deduced mathematically under Assumptions #1-3 as the prices existing in an efficient market where complete information about the real distribution of the relevant variables causes investors to hold positions truly maximizing their expected utility. Such true values have been shown to reflect higher discount rates on cash flows from assets characterized by greater systematic costs of trading as well as by higher contribution to an investor's risk such as measured by the return covariance with the aggregate market portfolio (Murphy, 1990), but the implications and intuitive richness of these modelled relationships and their associations with optimal trading behavior has not been previously examined.

The arbitragers defined in Assumption #4 would also have their trades determined by the equilibrium conditions defined in the Appendix, whereby their transactions would be a function of their total portfolio holdings. Such investors would include all but those engaging in strictly buy-and-hold strategies. Although some of these arbitragers might strictly hold positions designed to generate excess profits from their perceptions of mispricings defined by deviations from the values given under the equilibrium conditions of the Appendix, arbitrage transactions might also be undertaken at least occasionally by investors who allocate part or most of their portfolio holdings to buy-and-hold or even indexed strategies.

An arbitrage trade to exploit a deviation between market price and intrinsic value will currently occur only in certain circumstances. For instance, the net arbitrage profit derived over the next period from a correct transaction on an asset must exceed that available from any other investment to motivate a trade. Arbitrage might be especially unlikely to occur if an arbitrager perceives the chance of a near-term correction of a mispricing to be very low. Despite such realistic inhibitions on trading, the model explains the enormous transaction volume that does occur, unlike most models that have prices follow some pre-determined process.

Defining B as the full matrix of aggregate bids, net of the total selling orders, at any particular price in any period for a unit of a security, whose heterogeneously estimated value determines each investor's demand for and supply of the asset, the market clearing price for the asset will have to change anytime it results in a nonzero point in B that would imply imbalances between supply and demand. A positive (negative) value in B would require a rising (falling) price to attract more sellers (buyers) and discourage more bids (ask orders) until the orders balance at 0. In particular, when investors in the aggregate are motivated to purchase (dispose of) more units of an asset than the aggregate number desired to be liquidated (acquired) at a given price, the market must adjust to investors' weighted average valuation estimates to ensure equilibrium where net orders equal zero. The asset value estimates that determine the full distribution of B are a function of investors' time-varying heterogeneous utility functions, cash flow forecasts, and individualized valuation of relevant variables like taxes, transaction costs, and legal/moral constraints on their trading, as specified in the Appendix. Thus, the market price will follow a process that is determined by all these complex functions, which themselves are uncertain in nature.

In particular, the price of a security in any time period will tend to rise (fall) when the majority of new investors in the asset perceive the intrinsic value to be higher (lower) than exists in the market. The price will therefore rise (fall) toward this value whenever the new investors in the asset at the margin of trading in any period believe the asset is worth more (less) than that value. The resulting market prices will then reflect the weighted average value estimate of investors that are interactively and endogenously determined in an unspecified fashion.

For instance, the subjective perceptions of value that determine trading decisions may be affected in different ways by the biology-induced feelings that each investor can use at any moment in an individually customized attempt to optimally deal with the complexities involved in constructing a portfolio of investments in order to maximize their own heterogeneous utility functions under uncertain future conditions that they collectively affect interactively. As explained by Brennan and Lo (2011), their decisions may be impacted by both evolutionarily primitive and rational tendencies, which can even lead to changes in risk aversions levels being strictly based on whether an investment is perceived as a potential loss or opportunity. Investors' trades may therefore be influenced by both emotions and reason into unpredictable patterns which vary over time in a fashion that will only be determined by each individual investor in the future. The uncertain nature of the intrinsic valuation estimates of investors that vary both temporally and cross-sectionally therefore makes it impossible ex-ante to fully specify *B* exactly.

Although Sonnenschein (1973) has demonstrated that utility functions may be aggregated into some single equation proxy, it is necessary to know the individual preferences in order to be able to specify that function. In addition, Berg, Marsili, Rustichini, and Zechina (2001) have shown that, under several restrictive assumptions such as the existence of an equal number of informed and uninformed traders, markets will be more efficient the greater is the number of investors relative to the number of future possible states. However, since uncertainty essentially implies an indeterminate and potentially infinite number of states in the next period (that include an unlimited number of different future possibilities beyond one period hence for each given next-period market price), there can be no guarantee of any relationship between prices and intrinsic values whatsoever as long as there is only a finite number of investors. Gao, Song, and Wang (2013) have shown that, even if many of the relevant variable distributions and parameter values for investment analysis were somehow truly known, uncertainty in the number of informed traders alone can lead to multiple rational equilibria that can result in extreme price volatility and frequent detachment from intrinsic value.

The model's four assumptions thus permit trading, asset prices, and *D* to move in patterns that cannot be fully defined ex-ante through an objective probability distribution function with a finite number of variables.[1] Trends that emerge can therefore not be specified with certainty before they appear, nor can even the probability or existence of a pattern in the future be objectively known. In addition, it isn't possible to recognize if and when any particular relationship within a time series of prices might dissipate into complete randomness or even reverse in a contrarian fashion. It is also impossible to determine how many trades at any time will be made based on insightful decisions reflecting full information that would push an asset's price toward the true intrinsic value. Nevertheless, a framework for optimal decision-making by arbitragers who may utilize logical analysis and/or intuition to make their investment choices can be exactly defined within this environment of uncertainty. The implied decision-making criteria might ideally be used by an investor explicitly or implicitly.[2]

B. Mathematical Implications of the Four Model Assumptions

Assumptions #2 and #4 indicate that rational investors will not arbitrage a mispricing *D* on asset d as long as

$$\omega\,(D-\kappa) \leq \Psi. \tag{1}$$

The boundary condition (1) indicates that arbitraging trades will occur only if the direct cost Ψ of analyzing the particular asset is less than the arbitrage profit net of transaction costs $D-\kappa$ times ω, which is the probability an analysis will lead to a conclusion that an actual mispricing exists and should be exploited immediately.[3] In other words, a mispricing may persist unless it exceeds the sum of trading expenses and the total incremental costs investment analysis necessary to conduct a profitable transaction. Grossman and Stiglitz (1980) have previously shown that investors must be compensated for the direct costs of information, but rearranging (1) to solve the inequality for

D indicates the true total compensation to arbitragers net of transaction costs must equal Ψ/ω and not just Ψ.

Only if inequality (1) is violated will an asset even be analyzed. Thus, unless a perceived mispricing exceeds the sum of $\kappa + (\Psi/\omega)$, it will be ignored. If there is no reason to believe an asset is mispriced, then $\omega=0$, and the asset will not be considered, much less traded, as (1) would then hold even with an infinite mispricing. Since arbitragers' decisions to investigate and subsequently to possibly conduct transactions in an asset determine market prices, both *D* and the likelihood ω of a deviation between price and value actually existing are endogenous to the aggregated decision-making and trading of all investors. Roberts (1963) has indicated that people often use intuition to decide whether to obtain more information before making a decision, but (1) supplies a simple framework for analytically determining which assets have a high enough ω to optimally be evaluated.

The probability 1- ω of failed investigations helps explain the difficulty of obtaining venture capital and small business loans at all, much less on financing terms similar to those existing for companies that are widely recognized and evaluated. In order to attract any funding interest, securities of firms that are not well known must offer extra compensation that exceed the average costs incurred just to find such an investment with a price which deviates from value by more than the boundary condition (1). The extra required return of Ψ/ω would not exist on securities of firms which investors have already analyzed (such as in the case of large, well-established companies with publicly traded securities) and for which $\Psi/\omega = 0$.[4] The additional funding costs would be especially high if ω is small, as might be typical for newer businesses because of the large uncertainty on their intrinsic values that may be very difficult to evaluate. Uncertainty about value, and thus on *D* regardless of the actual price of the issues, stems not only from more limited knowledge about newer businesses and their future prospects but also from the potential for greater fraud and poor business management that might exist without an adequate track record.

The expenses Ψ/ω might be quite sizeable even for many publicly traded securities with small market capitalizations about which an investor knows little. Inequality (1) indicates that the latter investment possibilities are optimally neglected unless ω and *D* are sufficiently high to result in a violation of that boundary condition. In contrast, the incremental expense of investment analysis might approximate 0% for large-cap equities, which investment banks continuously analyze, and about which there is an abundance of easily accessible information.

With effective constraints on unlimited trading according to Assumption #4, arbitrage of d (and even an analysis thereof) still will not occur even with a violation of (1) after an investigation unless the expected net gain $D-\kappa$ from a particular arbitrage situation is no less than that available on another asset μ. A wider boundary for unarbitraged mispricings therefore arises, insofar as

$$D \leq (\Psi/\omega) + \kappa + M \qquad (2)$$

must hold. In a world with many assets, the boundary condition (2) would have to reflect the expected value of the largest net mispricing *M* on any other possible investment.

With a limited amount of arbitrage capital, multiple mispricings may inhibit the arbitrage that keeps prices from deviating from their intrinsic values. When systematic mispricings exist for many securities, the problem is exacerbated, since *M* may be magnified by larger mispricings on other assets. Thus, multiple mispricings may contribute to more mispricings, as might be especially characteristic of an overall market bubble or crash.

In addition, given the uncertainty of ω, it is entirely feasible for only a small group of arbitragers to correctly perceive a mispricing to exist at any moment. As a result, since Assumptions #1 and #2 allow for an unspecified number of investors to conduct trades that don't maximize their true utility under logical analysis given full information, there may be a predominate number of transactions that cause the full distribution of *B* to move prices even farther from their true intrinsic values. The potential increases in mispricings can give even knowledgeable investors an incentive to wait for larger deviations from perceived intrinsic values before engaging in arbitrage in order to be able to earn higher profits in the future. The very possibility that other arbitragers cognizant or "informed" of the true intrinsic value of a security that would exist in an efficient market will fail to conduct trades for the same or other reasons makes it even more likely that "uninformed" transactions will dominate in determining *B*, thereby raising the chance of increased mispricings.

To mathematically model this latter effect, the probability of sufficient transactions by arbitragers or other investors to remove the mispricing of d in the next period t+1 is defined as ρ, which is determined by an uncertain future distribution of potential orders at different prices that has values subjectively forecast heterogeneously by arbitragers for that period. Within this context, delineating optimal trader decision-making requires specification of a variable *Q*, which is defined to equal the product of the integrated sum of the expected risk-adjusted arbitrage profits (net of transaction costs), predicted to exist in period t+1 for each possible state of the world then, that is multiplied by the probability of the particular state occurring, conditional on no arbitrage having previously occurred. The state-dependent forecasted profits that determine *Q* represent the maximum net abnormal returns expected in each state of t+1 from arbitrage in any subsequent period t+n that themselves are a function of the predicted cash flows from d, as well as its future market prices, in all subsequent periods.

This definition for Q enables writing it mathematically as

$$Q=\underline{D}_{t+1}-\underline{\kappa}_{t+1}, \quad (3)$$

where $\underline{D}_{t+1}-\underline{\kappa}_{t+1}$ denotes an arbitrager's estimate of the weighted-average maximum expected value of the abnormal profit available on asset d across all states in period t+1, where this maximum in any state of period t+1 is the present value in that state of the highest income from arbitrage trading in any future period t+n>t (net of κ_{t+n} and Ψ_{t+n}), if d remains mispriced in period t+1. For simplicity of exposition and intuitive understanding of the effect on arbitrage decisions of the relevant factors, variables with values in the current period t are not subscripted.

Arbitrage will then not take place in asset d in the current period t as long as

$$D-(\Psi/\omega)-\kappa-M\leq(1-\rho)Q. \quad (4)$$

In particular, whenever an arbitrager perceives the probability ρ of other investors arbitraging the situation is low enough relative to the chance of a continuation of a trend in psychological optimism or pessimism that may be contributing to the maintenance of, or increase in, a mispricing, it can be more profitable to wait to trade.

In order to apply (4) computationally, a complete stream of *D* for which net orders in B_{t+n} equal zero must be predicted for each state in period t+1 in order to fully specify the value of *Q* in (3). Such forecasts theoretically should incorporate an entire distribution of predicted mispricings in each state of each future period t+n that is forecast to exist in each possible state of each subsequent period t+n-1 in order to compute the expected value of *D* in each state of period t+n. Each mispricing in that potentially infinite stream of different *D* could be affected by an infinite number of variables with uncertain values and distributions, thereby making precise calculation by a computer impossible. Since scientific studies indicating a combination of conscious human reasoning and intuition based on emotions are typically optimal in complex uncertain situations (Gigerenzer, 2007), the estimated values of ρ and *Q,* as well as *D* and *M,* might be rationally affected by biologically-induced emotions that would increase the subjectivity of investor decision-making.

Whether variable values are heterogeneously estimated by intuition or reason, the maximum size of a mispricing before it can be arbitraged away must be adjusted further upwards to

$$D \leq Q(1-\rho)+(\Psi/\omega)+\kappa+M, \tag{5}$$

which follows directly from rearranging (4).[5] The boundary condition (5) leads to many implications for investor behavior that can have a significant impact on market prices.

II. Specific Hypotheses Implied by the Model

As indicated in the prior section, boundary condition (5) indicates that the possible existence of an unknown and potentially very large mispricing in the future caused by an always uncertain number of misinformed investors can motivate informed investors to delay trying to exploit a mispricing even without overtly colluding. The likelihood of B reflecting misinformed investor trades is increased by such optimally motivated inactivity, thereby lowering ρ, raising Q, and enlarging the ceiling for D according to (5). The variable ρ and thus D are therefore impacted by factors other than the direct influence of misinformed investors, as knowledgeable investors affect ρ and the boundary condition for mispricings by their own estimates of ρ.

The probability of arbitrage is also reduced by past losses resulting from increases in mispricings that impair the transaction capacity of contrarian investors, who have conducted trades against a trend that may have even bankrupted some of them (Allen and Carletti, 2010). Such adverse price movements can also result in margin calls on past arbitrage positions that can lead to closing-out trades which then further increase mispricings (Thurner, Farmer, and Geanakoplos, 2012) as those transactions have the same effect as the trades of misinformed investors. In addition, the likelihood of arbitraging away a mispricing is further decreased by the mere risk of future losses (Abreu and Brunnermeier, 2002) that are specified in (5) by Q, which defines the actual variables determining the opportunity costs of conducting arbitrage in the next period. Predictions of such potential future trading costs are established endogenously in the model here by the valuation estimates of arbitragers and other investors that determine the full distribution of potential orders in B_{t+n}.

The endogenous and uncertain nature of ρ and Q allow those variables to affect each other across investors and thus impact D indirectly as well as directly in (5). For instance, an initially low (high) estimate for ρ (Q), perhaps based on past relationships, can realistically lead to beliefs that other investors applying (5) won't trade due to them believing yet other arbitragers also will wait to conduct a transaction. Such a prediction on the inactivity of other informed investors reduces ρ and thus makes it increasingly likely that D will increase instead of decrease in the future. In particular, the motivation for postponing arbitrage in expectation of better future opportunities may be raised any time arbitragers expect other knowledgeable investors to do the same in similar anticipation of even greater arbitrage profits being available to them in the future, i.e., when $Q>m$. Reduced arbitraging trades increase not only current mispricings but also the size of potential future deviations between value and price. As a result, ρ may be negatively correlated with Q, which in turn positively affects the maximum D according to (5). For instance, a drop in ρ may raise Q, thereby raising the potential mispricing according to that boundary condition, so that ρ may also be negatively correlated with D for reasons other than through its own direct relationship shown by (5).

Moreover, if an investor perceives the trading decisions of other arbitragers to be subject to the same type of psychological factors as for the typical human being, who might be inhibited from aggressive trading after suffering losses by natural biological factors (Murphy, 2012), it might be objectively concluded that ρ will be directly impacted downward by rising *D*. In particular, arbitragers might believe other investors may become psychologically discouraged from trading by increases in *D* that might naturally result in a biology-induced decline in their estimates of ρ. In addition, the increased investor risk aversion levels that result from past losses (Peterson, 2007) can further inhibit arbitrage and therefore also contribute to lowering ρ when *D* rises. Thus, rising mispricings can lead to arbitrager losses that sap not only their trading resources but also their willingness to commit any further capital, thereby intensifying any negative relationship between ρ and *D*.

With some investors sure of a mispricing potentially waiting for the possibility of larger arbitrage profits in the future, with other arbitragers failing to even investigate an opportunity because of a perception that an analysis has too small a chance of indicating a transaction should be undertaken, with yet others having insufficient capital to trade without losing the benefits of diversification from their perceived optimal portfolio holdings, with some investors indexing because they believe there are no mispricings that violate (1), and with an unknown number of investors trading in the opposite direction because of trends in confidence relating to normal biological tendencies as well as analytical error in valuation and margin calls, perceptions of the amount of informed trading incorporated into *B* can be quite small. The probability of market-correcting arbitrage ρ may therefore be very low. As shown by Abreu and Brunnermeier (2003), if ρ is small enough, arbitragers believing a mispricing might likely grow may even trade with a trend, thereby effectively resulting in increasing the recognized price deviation from intrinsic value.

Whenever the amount of investment money of informed investors about to trade is believed in any current or future period to be less than the funds of uninformed investors who are influenced by trend-inducing biological cycles or poor analysis to estimate the wrong sign for the difference between an asset's value and its price, ρ is driven toward zero, which can raise *Q* and *D* infinitely. With a mutually reinforcing, negative correlation of ρ with both *D* and *Q*, boundary condition (5) indicates mathematically that a circular negative feedback loop could exist that might increase the size of potential mispricings without limit. This very possibility may decrease ρ and increase *Q* further.

Some of these conclusions are similar to those indicated by Abreu and Brunnermeier (2003) under more restrictive conditions, which include assuming the existence of a known price process in the absence of arbitrage and the existence of a maximum for the duration of a mispricing trend. However, the uncertainties regarding the relative number of trades by investors misinformed about a mispricing, along with the uncertainty of sufficient arbitrage transactions at any point to offset or correct them, are indicated here to result in self-reinforcing trends that may persist indefinitely without any pre-determined distribution. While a rising mispricing might lead to sufficient interest and recognition of it to raise ρ at some unknown period in the future, thereby inhibiting infinite mispricings, the very possibility of an indefinite trend reduces ρ and raises *Q* further, thereby magnifying those trends and increasing their likelihood. While inequality (5) defines the boundary for the size of mispricings, values and relationships between the model variables can result in an ever increasing rise in a mispricing.

An introductory test of some of these hypotheses was conducted using data from 2/15/77 until the end of 2012 that indicated the probability of arbitrage was indeed negatively correlated with both current and future mispricings (i.e., with *D* and *Q*, respectively). In particular, confidence levels were estimated for the Center for Research in Security Prices (CRSP) value-weighted equity index by assuming a 5% equity premium above the yield on Treasury bonds maturing closest to 12/31/12 (obtained from the Federal Reserve website) and computing mispricings *D* by subtracting from the daily index value the present value of the dividend-reinvested CRSP index at the end of 2012. The expected value of future mispricings *Q* was specified as the maximum *D* on any future day until the mispricing was reversed in sign. The probability ρ of arbitrage was set equal to 1 (0) if a mispricing is fully corrected (becomes larger afterwards without correcting) on the next day and to the percentage reduction in the mispricing on the next day otherwise. Plugging these values into (5), which is set as an equation, enables solving for the confidence level ρ of traders at the margin, i.e., of investors who are on the verge of trading. The results indicated that ρ, which had a mean of .02 and a standard deviation of .10, exhibited a correlation of -.20 with *D* and -.24 with *Q*.

These empirical findings are consistent with a typical lack of urgency in conducting arbitrage trades in practice, as there tends to be only a very small chance a mispricing will disappear in the future. The boundary condition (5) indicates mispricings can be quite large under these circumstances. The negative relationship found here between ρ and *D* as well as *Q* supplies evidence in favor of the previously explained reasons why trends in mispricings persist and grow. On the other hand, the fact that the negative correlation between those variables is far below -1.00 implies that there are finite, albeit uncertain, limits to the latter destabilizing effect.

A. Market-Wide Impacts

The same factors influencing *D* would also impact *M*. For instance, trends in psychological tendencies induced by biological factors like systematically varying testosterone levels among investors could actually make $Q>D-\kappa-\{\Psi/\omega\}$ for virtually all assets due to the trading patterns induced by the resulting typical mental states, thus leading to (5) holding systematically in the market. If there are no violations of (5) for any asset, i.e., if arbitragers perceive the probability of a sufficient widening in the mispricing *M* to be large enough to make it more profitable to wait to arbitrage that asset as well as d, arbitragers would optimally invest in risk-free assets f. The possibility that many informed investors will not make any arbitraging transactions in d or μ increases the potential size of future mispricings *Q* on both assets because the trades of uninformed investors are then more likely to dominate both assets' prices and increase both *D* and *M*.[6]

Also potentially contributing to a reduction in ρ with rises in *M* are the decreases in available capital for diversified investors when market mispricings increase, as would be especially likely in the case of widespread underpricings characterizing bear markets which reduce the value of investors' positions in the aggregate that have to be net long to absorb the supply of existing assets. These phenomena that inhibit arbitrage activity reduce ρ when *M* rises, thereby increasing the potential value of *D* according to (5). Thus, the rises in *M* can decrease *D* both directly in (5) as well as indirectly by lowering ρ in a fashion that further increases *D* and the mispricing on other securities as well. This circular relationship makes bubbles and crashes in the overall market very feasible.

B. Effects on Specific types of Assets

The foregoing market-wide impacts may be larger on the securities of smaller firms. In particular, inequality (5) indicates that the mispricing level necessary to trigger an arbitrage trade is higher for securities whose proportional costs of trading κ are greater, as may be characteristic of small-cap securities. The fact that large sophisticated traders are subject to especially high transaction costs on small-cap stocks (Keim and Madhaven, 1998) exacerbates this problem for these assets, since such expenses alone inhibit the arbitrage of those investors who might normally be more skilled recognizing if an asset is mispriced or not.

In addition, the securities of smaller firms tend to be more costly to analyze per dollar of the aggregate arbitrage trade possible in these assets that have total market values which are low relative to the portfolio size of the largest institutional investors. These higher analytical costs cause a higher Ψ for such arbitragers because the largely fixed cost of analysis for each such asset is spread over a smaller market value compared to large-cap securities. This phenomenon results in assets with small market capitalizations not normally justifying the costs of continuous, in-depth analytical attention by institutional investors because the total profit possible for arbitraging a given percentage mispricing *D* is lower than for large cap securities (Merton, 1987). An investment analysis of such firms would therefore require more incremental fixed costs, thus raising Ψ even more. The boundary condition (1) alone indicates such sophisticated investors would therefore be less likely to ever consider such investments, thus reducing ρ and raising the maximum *D* further.

All these phenomena result in the maximum mispricing in (5) for such assets to be higher. With a larger possible *D*, the prices of such assets might be expected to move more freely in systematic trends of biology-induced sentiment that can increase the mispricings on other assets (and thus *M*) at the same time.

With the mispricings of all assets often tending to move in the same direction because of the directional relationship shown in (5) as well as because of trends in market sentiment inducing a positive covariance between *M* and *D*, albeit not as strongly as for small-cap issues, the mispricings of the securities from smaller companies would co-vary more with those of others. A higher covariance between *D* and *M* implies higher systematic risk for such investments because they might, for the aforementioned reasons, suffer greater losses in overall bear markets induced by prices generally falling below their intrinsic values then.

This impact on market co-movements for more illiquid securities would be exacerbated during liquidity crises relating to declines in market prices because the costs of trading those assets may rise relatively more at that time. In particular, a larger *M* that contributes to magnifying mispricings on d during market busts may often happen at the same time as transaction costs κ rise. For instance, in a market crash, many investors are desperate for liquidity to cover margin calls and other cash needs resulting from both declining wealth (Thurner, Farmer, and Geanakoplos, 2012) and falling risk tolerance levels, which Peterson (2007) has shown tends to decrease after losses. Investors' transactions to raise money in market crises have been found to be most heavily focused on more liquid securities to minimize their trading expenses, thereby decreasing the relative volume and raising transaction expenses of other assets in times of extreme demand for cash (Goyenko, Subrahmanyam, and Ukhov, 2011). Any relatively larger increase in κ caused by rising *M* would therefore be concentrated on small-cap securities in market extremes, thus further magnifying the multiplicative relationship between *M* and *D* for those assets in such environments according to (5).

Moreover, rapidly moving prices might also magnify Ψ on the securities of smaller firms. In particular, the time required to analyze and urgently appraise their values might become prohibitively high when there are the large price changes typical of a crashing market. The opportunity cost of analysis might likely rise then due to the existence of more lucrative arbitrage opportunities that might quickly disappear in the volatile market, and so the effective expense per minute of investigative analysis can rise substantially at that time. The resulting even higher differences in expected costs Ψ on assets with lower market capitalizations during fast market declines would increase the potential size of *D* for those assets according to (5), thus enabling their unarbitraged mispricings *D* to move more with other assets *M*.

In addition, the likely larger increase in κ and Ψ on small-cap issues during future bear markets would raise *Q*. The magnitude of a mispricing needed to trigger arbitrage might therefore increase yet more according to (5), thereby contributing further to their greater systematic risk which would be maximized during widespread price declines.

Such greater contribution to the risk of diversified portfolios for all small cap issues might be especially pronounced in cases of very large market-wide mispricings that are possible according to (5) but seldom if ever observed empirically. As a result, the full extent of the greater covariance of the returns on small-cap securities with those of the market isn't normally estimated from simple examination of past finite samples, even though higher returns may be required for such unobserved systematic risk. Since such risk may be underestimated in traditional empirical studies because the possibility of exceptionally large losses during market extremes on the downside may never have occurred in the past, research that ignores this phenomenon may conclude higher returns on small-cap stocks to represent an anomaly. In particular, although some extreme events that have a large impact on market prices are observed in very long-term samples (Barro, 2006), others remain mere possibilities in an uncertain world and may thus lead to perceptions of different risk than past history would indicate. The possibility of extremes that have been rarely if ever experienced but are quite feasible have been empirically shown to explain market anomalies with respect to both foreign exchange rates (Murphy and Zhu, 2008) and stock returns (Bollerlev and Todorov, 2011).

The small firm effect associated with equities having lower market capitalizations demonstrating higher estimated risk-adjusted returns in empirical samples (Stoll and Whaley, 1982) could be fully explained by such phenomena. Higher returns have also been empirically found on stocks more ignored by professional investors (Richardson, Sloan, and You, 2012). A portion of those higher returns could merely represent compensation for the portion of those assets' higher systematic risk that isn't typically measured.

The impact that stronger systematic movements would have on required returns is over and above the neglected firm effect that Merton (1987) has shown, under some restrictive assumptions, can lead to higher returns on more ignored assets that are held in less diversified portfolios and thus have priced unsystematic risk. The higher systematic risk premiums on small-cap securities combined with the extra return required for the full expected value of the costs Ψ/ω of investigating those assets may alone be sufficient to fully explain the neglected and small firm effects.

C. The Influence of Institutional Investor Incentives

The performance bonuses for institutional investors that are usually set as a percentage of the accumulated trading profits by the end of the fiscal year may also affect ρ and thereby cause other patterns in the market place. For example, towards the end of the year that terminates in December for many institutions, managers are less likely to invest in risky assets since losses then are more likely to have a negative effect on the expected value of the accrued managerial bonuses, which are typically set as a function of total gains during the year and restricted to be non-negative (Coates, 2012). In particular, such compensation schemes create a put with an effective strike price at 0 on manager return participation, and this option tends to have a lower value toward the end of the year because the typically larger accumulated positive returns by then make the option more in the money. Institutional investor aversion to risk might therefore usually increase as the months pass in a process that might begin to have an especially significant effect by August of each year when many managers take time off for summer vacations (Coates, 2012). This impact may be especially strong in years with rising markets that would generate more accumulated profits for traders to protect in the second half of the year by taking less risk until the start of the following fiscal year. As a result of decreases in investor risk-taking at that time, there might be a reduction in ρ then, as well as a decline in the market prices of risky assets in general during the second half of the year. The empirical findings of lower stock returns in the second half of the year (Ziemba, 2012) are consistent with this hypothesis.

In contrast, the occurrence of non-positive returns since the start of the year in January may motivate riskier trading in the second half of the fiscal year because investing after such occurrences (that are less frequent than positive returns but do happen) has a higher expected value of manager pay due to additional losses not reducing compensation set as a percent of investment profits any more than total inactivity. As a result of bonuses being zero regardless of whether there is no change in negative returns or an increase in a net loss, ρ may actually be increased after the spring if market returns have been low or negative in the year's prior months.

With the start of each new year, the incentive to take risk might increase significantly. For instance, given that the fiscal year end for many mutual funds is October (Amman and Verholfen, 2007), an inhibition on arbitrage trading after large gains might only last through that fall month. In particular, increased risk-taking would be promoted in fund managers at the beginning of their new performance measurement period in November when losses don't take away as much from the expected value of end-of-year bonuses compared to later months when accumulated trading profits might be greater. The result of annual earnings typically not being as high early in a new year as later might cause returns on risky assets being highest in January because it is only 2 months into the fiscal year for the investment funds and the actual start of the year for other institutional investors. The January effect of higher excess returns in that month than at any other time (Tinic and West, 1984) may be partially explained by this institutional characteristic of greater risk-taking at the beginning of the year that stem from the put option on bonuses which typically have a $0 floor.

Higher returns and accumulated bonuses early in the year might prompt the production of optimism-inducing testosterone that can motivate more aggressive trading in later calendar months (Coates, 2012). The January barometer indicating that stock returns in the initial month of the year are positively correlated with those over the remaining eleven months (Ziemba, 2012) is consistent with this hypothesis. This psychological impact appears to more than offset the rationally inhibiting effect associated with protecting the performance bonus in the second half of bull market years

Perhaps because of the possible conflicting influences of these factors, Ammann and Verhofen (2007) have cited studies indicating the empirical evidence on the motivational factors involved with the time of the fiscal year is mixed to date. However, in their own study, they did discover that managers take more risk following successful performance in the prior year, as is consistent with a hypothesis that the confidence of professional investors is affected by human biology that promotes trends in optimism. This psychological influence may have a separate impact on p and hence on D.

All these various relationships that can affect mispricings D can't be reliably predicted mechanically because other investors' estimates of the probability ρ of a market correction of a mispricing can incorporate such forecasts as well as any other factors, thereby affecting ρ in potentially unpredictable ways. While this endogenous nature of ρ makes uncertainty in the markets inevitable, (5) enables a concrete analysis of the relationships that can increase the effectiveness of investment management.[7]

III. Conclusion

This research develops a normative model for optimal investment decision-making under the realistic conditions of considerations endogenous to investor psychology and uncertainty that tend to result in deviations between market prices and intrinsic values. In particular, a simple boundary condition necessary to trigger arbitrage is analytically derived that may be utilized by fundamental analysts to determine optimal entry and exit points, as well as be employed by technicians seeking to profit from patterns observed in market variables. The framework provides a useful stepping stone in integrating logical investment analysis with human biology, intuition, and uncertainty.[8]

The model facilitates a deeper understanding of how financial cycles can exist without being arbitragable. In particular, endogenously determined uncertainty as to the likelihood of a market correction of a mispricing enables incorporating the psychological issues that can contribute to increasing mispricings which may lead to financial bubbles and crashes even when markets are fundamentally efficient. For instance, biologically-induced systematic changes in investors' psychology are directly integrated into the model of rational arbitrager behavior in the unspecified fashion that promotes market prices which are both cyclical and unpredictable. It thereby clarifies why there can be unarbitrageable patterns such as those spurred by past trading successes that lead to aggregate investor overconfidence and booms, which may be followed by sharp reversals after sufficient investment failure. In particular, trends may persist until mispricings become large enough to make the deviations between prices and values clear enough to arbitragers to motivate a number of informed trades sufficient to dominate the effect of uninformed investors and push prices toward values.

The modeling foundation here supplies a more realistic perspective for evaluations of the efficiency of any market in reflecting value and may help explain some observed market anomalies. For instance, it provides a framework for understanding how the compensation schemes of institutional investors can create seasonal patterns such as the January effect. In addition, the analysis of investor and arbitrager psychological tendencies lays the groundwork for a better comprehension of the differential systematic risk and return of small-cap stocks, which may move more with overall market mispricings due to higher costs of trading and evaluation. Moreover, the model decision-making process for arbitragers that integrates the determination of whether to access and process more information on an asset enables incorporating the full cost of investment analysis into trading decisions and helps explain the neglected firm effect.

Footnotes

1. Using numerous simplifying assumptions such as the existence of a fixed number of fundamentalists who are always certain of the exact intrinsic value of any asset and trade immediately upon any observed mispricing, Lux and Marches (1999) developed a model that results in price patterns that resemble those observed empirically. Simply postulating the mass behavior of crowds of investors to be similar to the complex chaos of inanimate phenomena such as the weather and earthquakes, actual patterns in financial prices can also be replicated (Buchanan, 2013). However, any modeling of or distributional assumptions about the behavior of investors in the future are intrinsically divorced from reality because investor decision-making is endogenous to trading and market prices. For instance, even if past patterns in investor behavior and prices have consistently repeated themselves, investors are likely to eventually react to observed market relationships and thereby change the patterns themselves in typically unpredictable ways (Murphy, 2012). Mathematical or statistical models may be reliable indicators of natural phenomena that can't consciously react to the patterns they cause, but they may be less accurate in describing/forecasting the behavior of human beings whose actions do reflect their knowledge about such models and prior patterns (Buchanan, 2013).
2. Despite the eloquence of many models that have been derived various different assumptions that include the fairly unrestricted conditions utilized by Murphy (1990) to derive complex equilibrium equations, specific criteria yielding important implications on market behavior under real-world conditions represent a potentially revolutionary advance in analyzing business choices in general. For instance, it has been found that business decisions based strictly on rational analysis not only lead to typically suboptimal choices but also take an excessive amount of time (Gigerenzer, 2007). To avoid such problems, the very decision as to the optimal combination of intuition and conscious analysis to employ in a decision is usually made intuitively. However, it is possible to specify a rule to optimize this process as well as to determine the optimality of gathering and processing more information. In particular, a rational investigation should optimally be initiated or continued anytime the cost of the analysis is less than the expected value of the increase in the abnormal profit resulting from the investigation leading to optimally determining if a position should be taken on d, μ, or f.
3. Note that equation (1) implies that some amount of analysis will enable certain determination of whether an asset is mispriced or not. Such certainty, which might exist when defining an asset's intrinsic value as that determined in a semi-strong efficient market where all public information about an asset is obtained and optimally analyzed (Fama, 1970), is consistent with the level of confidence that professional money managers have in their valuation estimates (Russo and Schoemaker, 1992). However, both the assumptions and equation (1) allow for the possibility that it isn't actually feasible to determine whether an asset is mispriced or not with 100% certainty because Ψ can be infinite. In the latter case, there would be no boundary on mispricings.
4. Because the securities being offered have to be held net long (and can't effectively be shorted prior to any potential issue), the extra return would have to be to the long positions. As a result, this extra "mispricing" relative to firms with Ψ=0 would have to

represent an increase in the required return on the assets for buyers, thereby leading to a higher discount rate on the cash flows and hence a lower value than would exist in a world with perfect information.

5. Note that initial estimates for the variables in (4), such as obtained from a computer program or screening process, can enable decision-making without an investigation, if the direction of a potential mispricing is known with certainty. Even if only the difference between the likelihood of a mispricing in one direction and the other can be estimated, (4) can still be utilized as a trigger point for a trade without further investigation if D, κ, and Q are first multiplied by that probability (with the same process also being undertaken for all other assets to determine the maximum M). However, great uncertainty about a mispricing without an investigation makes trades based on less than a fully valid analysis of all information have an unknown risk of an incorrect estimate for both the sign and size of a mispricing, thereby resulting in a potentially prohibitive risk of very large losses when the true mispricing (that could be infinite as explained in footnote #3) is corrected. A good example of the latter with respect to utilizing computer software to make investment decisions without a full valuation analysis may be provided by both the credit bubble and subsequent crisis (Murphy, 2010) as well as by the "quant" crash of August 2007 (Buchanan, 2013). Regardless, investors utilizing (4) without complete information don't actually set a boundary condition for the true maximum value of D in (5) because their transactions, which aren't based on completely informed valuation estimates, may therefore actually magnify as opposed to eliminate a mispricing.
6. It should be mentioned that even arbitrage trading of assets with the largest total profit potential might not drive prices toward value if there are multiple mispricings. In particular, the very uncertainty of mispricings being arbitraged away near-term can cause investors to simultaneously take positions in several mispriced assets that combined earn abnormally high portfolio returns regardless of whether prices move toward or away from intrinsic value. A good example is provided by the arbitrage of the subprime mortgage market bubble by the hedge fund Magnetar, which received income through leveraged investments into higher-risk home loans that it deliberately had originated to package into pools for sale to other investors, and which simultaneously used a portion of that income to buy excess protection via credit default swaps (CDSs) upon which it profited even more when the market collapsed (Eisinger and Bernstein, 2010). Because both the mortgages and the CDSs were mispriced in an interrelated but not identical fashion (Murphy, 2010), Magnetar was able to earn hedged arbitrage permits profits regardless of the direction of the market, albeit without correcting the overall level of mispricings. This strategy may have actually raised the degree of market inefficiency by increasing the supply of new mispriced assets (i.e., the additional subprime mortgages in their pools that might otherwise have never been originated).
7. Although Manski (2004) has precisely modeled rational decision-making within the context of expectations about a single dichotomous choice of another rational agent, (5) provides the framework for determining optimal actions when there are many heterogeneous investors who interactively and potentially irrationally affect investment outcomes over multiple time periods.

8. Models such as by Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010) and Oruguz (2009) indicating a higher return required on investments with ambiguous outcomes or during periods of greater uncertainty may also be enhanced by utilizing the generalized framework developed here that doesn't impose any restrictive assumptions on the mispricing of assets. For example, the precise decision-making framework of this research enables understanding the pricing of different assets with uncertain payoffs that aren't fully specified and can't be hedged. In addition, by enabling a better comprehension of the factors affecting the unknown future distribution of deviations between price and value, use of the boundary condition (5) to determine if a trade should be conducted may even reduce the perceived level of uncertainty for such investors, thereby lowering both their aversion to it and the premium required to compensate for it.

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